Section 3.4 Curve sketching (Minimum Homework: all odds)

We will learn how to sketch graphs of functions in this section. There are a few skills from College Algebra that we need to recall.

- Finding x-intercepts of the graph of a function.
- Finding y-intercepts of the graph of a function.
- Finding the equation of a vertical asymptote in the graph of a rational (fraction) function.
- Finding the equation of a horizontal asymptote in the graph of a rational (fraction) function.

Find the x-intercepts for the graphs of: (set function equal to zero and solve for x)

- $f(x) = x^3 - 9x$
- $f(x) = \frac{x-5}{x+6}$
- $f(x) = 7xe^x$

$x - intercept\ of\quad f(x) = x^3 - 9x$

$x^3 - 9x = 0$
$x(x^2 - 9) = 0$
$x(x + 3)(x - 3) = 0$
$x = 0 \quad x + 3 = 0 \quad x - 3 = 0$
$x = 0 \qquad x = -3 \qquad x = 3$

$x - intercepts\ \ (0,0)\ (-3,0)\ \ (3,0)$

$x - intercpet\ of\ f(x) = \frac{x-5}{x+6}$

Shortcut is to set the numerator equal to zero and solve. This Algebra gives the x-coordinate of the x-intercept, provided the value of x is in the domain of the function.

$x - 5 = 0$
$x = 5$

$x - intercept\ \ (5,0)$

$x - intecept\ of\ \ f(x) = 7xe^{x}$

$7xe^{x} = 0$

$7x = 0 \qquad\qquad e^{x} = 0$

$x = \frac{0}{7} = 0 \qquad no\ solution\ to\ e^{x} = 0$

$x - intercept\ \ (0,0)$

Find the y-intercepts for the graphs of: (substitute $x = 0$)

- $f(x) = x^3 - 9x$
- $f(x) = \frac{x-5}{x+6}$
- $f(x) = 7xe^x$

$y - intercept\ f(x) = x^3 - 9x$

$f(x) = (0)^3 - 9(0) = 0$

$y - intercept\ \ (0,0)$

$y - intercept\ \ f(x) = \frac{x-5}{x+6}$

$f(0) = \frac{0-5}{0+6} = -\frac{5}{6}$

$y - intercept\ \ \left(0, -\frac{5}{6}\right)$

$y - intercept\ \ f(x) = 7xe^x$

$f(0) = 7(0)e^0 = 7(0)(1) = 0$

$y - intercept\ \ (0,0)$

Find the vertical asymptote of the graphs of:

- $f(x) = \frac{7}{x-4}$
- $f(x) = \frac{x-5}{x+6}$

A rational function as a vertical asymptote with equation $x = \#$ for all values of x that:

- Make the denominator equal to zero
- Come from factors that do not cancel with the numerator

To find a vertical asymptote of a fraction simply set the denominator equal to zero and solve for x. Check to make sure the factor that creates the vertical asymptote does not cancel with a factor in the numerator.

Find the vertical asymptote of the graph of $f(x) = \frac{7}{x-4}$

$x - 4 = 0$

$x = 4$

Note: The factor of $x - 4$ does not cancel with the numerator.

$vertical\ asymptote\ \ x = 4$

Find the vertical asymptote of the graph of $f(x) = \frac{x-5}{x+6}$

$x + 6 = 0$

$x = -6$

Note: The factor of $x + 6$ does not cancel with the numerator.

$vertical\ asymptote\ \ x = -6$

Shortcut to find Horizontal Asymptotes:

$$f(x) = \frac{smaller\ degree}{larger\ degree} \qquad y = 0$$

$$f(x) = \frac{same\ degree}{same\ degree} \qquad y = \frac{leading\ coefficient}{leading\ coefficient}$$

$$f(x) = \frac{larger\ degree}{smaller\ degree} \qquad \text{No Horizontal Asymtote}$$

Find the horizontal asymptote of the graphs of:

- $f(x) = \frac{7}{x-4}$
- $f(x) = \frac{x-5}{x+6}$

Find the horizontal asymptote of the graphs of $f(x) = \frac{7}{x-4}$

The numerator is degree 0
The denominator is degree 1

This is the top case above:

$$horizontal\ asymptote\ \ y = 0\ \ (this\ is\ the\ x - axis)$$

Find the horizontal asymptote of the graphs of $f(x) = \frac{1x-5}{1x+6}$

The numerator is degree 1
The denominator is degree 1

This is the middle case above:

Divide the coefficients of the x's to find the horizontal asymptote.

$$y = \frac{1}{1} = 1$$

$$horizontal\ asymptote:\ y = 1$$

I will type up solutions to 2 of even numbered problems in the section. There are handwritten solutions for all the odd numbered problems under the solutions tab.

Problem

2) $f(x) = 2x^3 - 12x^2$

a) Find the x-intercept(s), if any

$2x^3 - 12x^2 = 0$
$2x^2(x - 6) = 0$
$2x^2 = 0 \qquad\qquad x - 6 = 0$
$x^2 = \frac{0}{2} \qquad\qquad x = 6$
$x^2 = 0$
$x = \sqrt{0}$ *no plus or minus needed when taking square root of* 0
$x = 0$

$x - intercepts \quad (0,0)\ and\ (6,0)$
Plot these two points on the x-axis

b) Find the y-intercept, in there is one

$f(0) = 2(0)^3 - 12(0)^2 = 0$

$y - intercept \quad (0,0)$
Plot this point on the y-axis.

c) Find the interval(s) where the graph of the function is increasing
d) Find the interval(s) where the graph of the function is decreasing
e) Find all relative maxima
f) Find all relative minima

$c, d, e \text{ and } f \text{ can be done in one long step}$

1) Find the derivative of the given function.

$$f'(x) = 6x^2 - 24x$$

2) Find the critical numbers for the derivative.

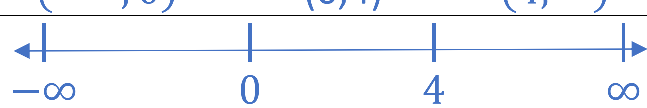
$6x^2 - 24x = 0$
$6x(x - 4) = 0$
$6x = 0 \qquad x - 4 = 0$
$x = 0 \qquad x = 4$

Critical numbers $x = 0, 4$

3) Plot the critical numbers on a number line that also includes $-\infty \text{ and } \infty$.

4) Create interval(s) using only round parenthesis.

$(-\infty, 0)$	$(0,4)$	$(4, \infty)$



5) Pick a number inside the interval and plug it into the derivative.

$(-\infty, 0)$ $choose\ x = -1$

$f'(-1) = 6(-1)^2 - 24(-1) = 30$

$(0,4)$ $choose\ x = 1$

$f'(1) = 6(1)^2 - 24(1) = -18$

$(4, \infty)$ $choose\ x = 5$

$f'(5) = 6(5)^2 - 24(5) = 30$

6) Determine whether the graph is increasing or decreasing in that interval.

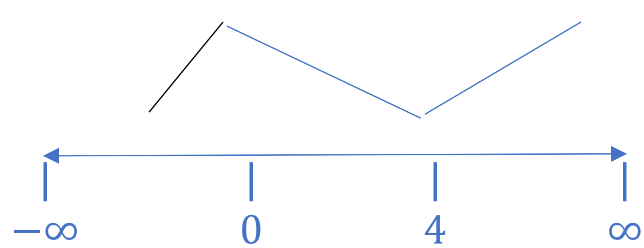
$(-\infty, 0)$ positive - increasing

$(0,4)$ negative - decreasing

$(4, \infty)$ positive - increasing

7) Place directional arrows in each interval to signify whether the graph in increasing or decreasing.

- Determine if each critical point is a relative maximum point, a relative minimum point, or neither.



Relative maximum point at $x = 0$

Relative minimum point at $x = 4$

8) Find the y-coordinates of any relative maximum or relative minimum points.

y-coordinate of relative maximum: $y = f(0) = 2(0)^3 - 12(0)^2 = 0$

relative maximum point $(0,0)$

y-coordinate of relative minimum: $y = f(4) = 2(4)^3 - 24(4) = -64$

relative minimum point $(4, -64)$

$Plot\ the\ points\ (0,0)\ and\ (4, -64)$

Answers:
c) Find the interval(s) where the graph of the function is increasing
$(-\infty, 0) \cup (4, \infty)$
d) Find the interval(s) where the graph of the function is decreasing
$(0,4)$
e) Find all relative maxima $(0,0)$
f) Find all relative minima $(4, -64)$

g) Find the interval(s) where the graph of the function is concave up (if any)

h) Find the interval(s) where the graph of the function is concave down (if any)

i) Find all inflection points (if any)

g, h and i can be done in one long step

Steps:

1) Find the second derivative of the function.

$f'(x) = 6x^2 - 24x$
$f''(x) = 12x - 24$

2) Find all critical numbers for the second derivative.

$12x - 24 = 0$
$12x = 24$
Critical number $x = 2$

3) Plot the critical number(s) on a number line that also includes $-\infty$ *and* ∞.

4) Create interval(s) using only round parenthesis.

	$(-\infty, 2)$	$(2, \infty)$
$-\infty$ — 2 — ∞		

5) Pick a number inside the interval and plug it into the derivative.

Interval: $(-\infty, 2)$
Check $x = 0$
$f''(0) = 12(0) - 24 \; = \; -24$

Interval: $(2, \infty)$
Check $x = 3$
$f''(3) = 12(3) - 24 \; = \; 12$

6) Determine whether the graph is concave up or concave down in that interval.
Interval: $(-\infty, 2)$ *concave down since f'' is negative*
Interval $(2, \infty)$ *concave up since f'' is positive*

7) Determine if the critical point is an inflection point. A critical number is an inflection point when:

The graph changes concavity at $x = 2$ and the graph is continuous at $x = 2$.
$x = 2$ is the x-coordinate of an inflection point.

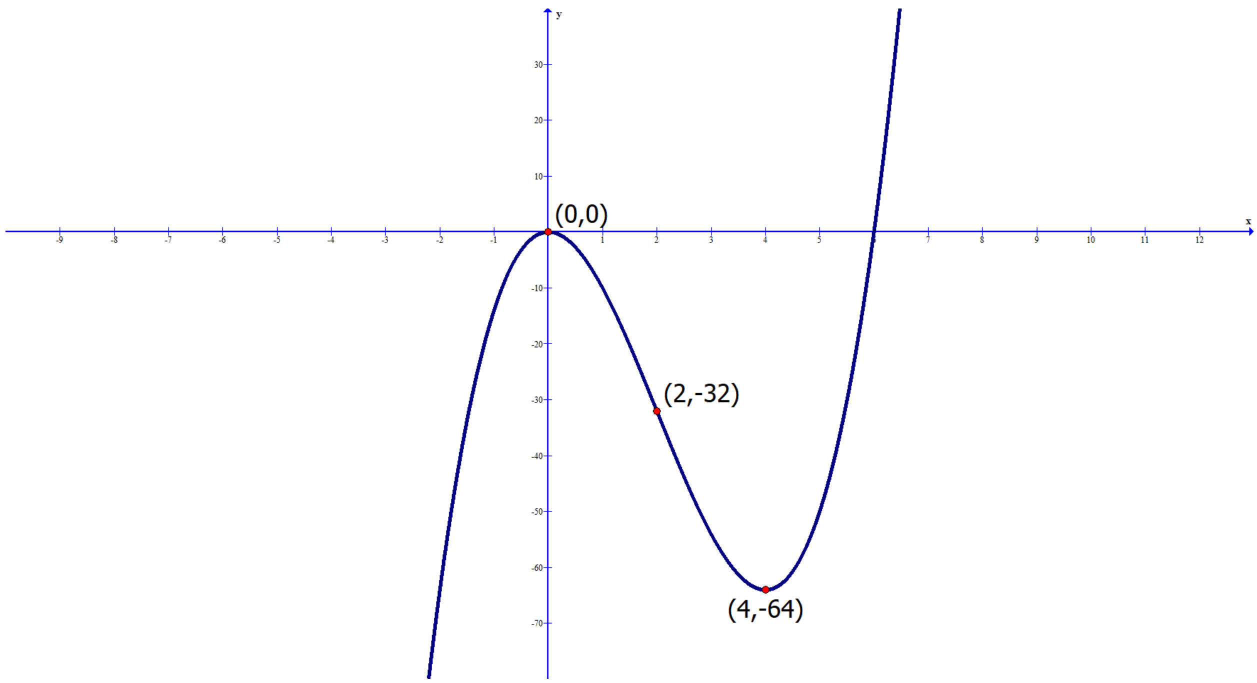
8) Find the y-coordinates of any inflection points.

$y = f(2) = 2(2)^3 - 12(2)^2 = \; -32$

Inflection point $(2, -32)$

g) Find the open interval(s) where the function is concave up $(2, \infty)$
h) Find the open interval(s) where the graph of the function is concave down. $(-\infty, 2)$
i) Find all inflection points $(2, -32)$

j) Sketch a graph: Plot all the points and try to make your graph increase / decrease in the correct intervals, and make sure the concavity matches the answers.



8) $f(x) = \frac{2}{x-3}$ Hint: $f"(x) = \frac{-2}{(x-3)^2}$ $f"(x) = \frac{4}{(x-3)^3}$

a) Find the domain

b) Find the equation of the vertical asymptote

c) Find the x-intercept(s), if any

d) Find the y-intercept, in there is one

e) Find all horizontal asymptotes

f) Find the interval(s) where the graph of the function is increasing

g) Find the interval(s) where the graph of the function is decreasing

h) Find all relative maxima

i) Find all relative minima

j) Find the interval(s) where the graph of the function is concave up (if any)

k) Find the interval(s) where the graph of the function is concave down (if any)

l) Find all inflection points (if any)

m) Sketch a graph

a)

$x - 3 = 0$

$x = 3$

$domain\ \ (-\infty, 3) \cup$
$(3, \infty)\ \ \ or\ you\ may\ write\ domain\ is\ all\ real\ number\ except\ x = 3$

b)

$vertical\ asymptote{:}\ \ \ x = 3$
$(x - 3\ is\ the\ factor\ that\ makes\ the\ denominator\ equal\ to\ zero,$
$x - 3\ does\ not\ cancel\ with\ the\ numerator.)$

Draw a dashed vertical line through 3 on the x-axis.

c) $x - intercept$

$2 = 0$

There is no solution as there is no x.

$no\ x - interept\ \ (graph\ will\ not\ touch\ nor\ cross\ the\ x - axis.$

d) $y - intercept$

$f(0) = \frac{2}{0-3} = -2/3$

$y - intercept\ (\ 0, -2/3)$

Plot the point $\left(0, -\frac{2}{3}\right)$ $on\ the\ y - axis\ just\ below\ the\ origin$

e) $horizontal\ asymptote$

This is the top of the three cases, no algebra for this.

$horzontal\ asymptote\ \ y = 0$
draw a dashed line on the x-axis to represent the horizontal asymptote.

f) Find the interval(s) where the graph of the function is increasing

g) Find the interval(s) where the graph of the function is decreasing

h) Find all relative maxima and

i) Find all relative minima

One long computation for f, g, h, i

1) Find the derivative of the given function.

$$f'(x) = \frac{-2}{(x-3)^2}$$

2) Find the critical numbers for the derivative.

$-2 = 0$ (no solution)

$(x-3)^2 = 0$

$x - 3 = \sqrt{0}\ no \pm needed\ when\ square\ rooting\ 0$

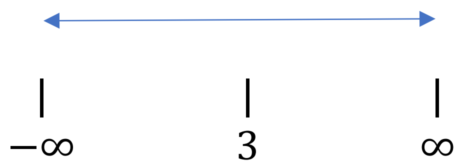
$x - 3 = 0$

$x = 3$

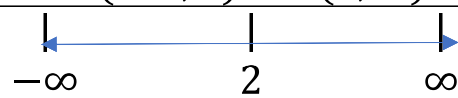
Critical number $x = 3$

3) Plot the critical numbers on a number line that also includes $-\infty$ *and* ∞.

Plot the critical number(s) on a number line that also includes $-\infty$ *and* ∞.



4) Create interval(s) using only round parenthesis.

$(-\infty, 3)$ $(3, \infty)$


5) Pick a number inside the interval and plug it into the derivative.

$(-\infty, 3)$ *choose* $x = 0$

$$f'(0) = \frac{-2}{(0-3)^2} = -\frac{2}{9}\ (negative\ number)$$

$(3, \infty)$ *choose* $x = 4$

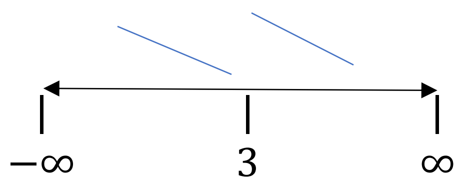
$$f'(4) = \frac{-4}{(4-3)^2} = -4\ (negative\ number)$$

6) Determine whether the graph is increasing or decreasing in that interval.

$(-\infty, 3)$ negative - decreasing

$(3, \infty)$ negative - decreasing

7) Place directional arrows in each interval to signify whether the graph in increasing or decreasing.



NO max NOR min

8) Find the y-coordinates of any relative maximum or relative minimum points.

NO work needed, as there are neither a max nor a min.

f) Find the interval(s) where the graph of the function is increasing
Never

g) Find the interval(s) where the graph of the function is decreasing
$(-\infty, 3) \cup (3, \infty)$

h) Find all relative maxima None

i) Find all relative minima None

One long computation for j, k, and i

j) Find the interval(s) where the graph of the function is concave up (if any)

k) Find the interval(s) where the graph of the function is concave down (if any)

l) Find all inflection points (if any)

Steps:

1) Find the second derivative of the function.

$$f''(x) = \frac{4}{(x-3)^3}$$

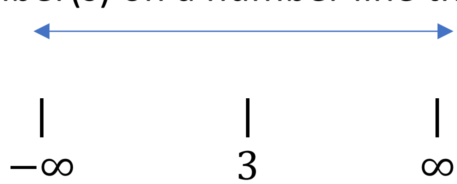
2) Find all critical numbers for the second derivative.

$4 =$
0 *no solution to this so no critical number from this calculation*

$(x-3)^3 = 0$
$x - 3 = \sqrt[3]{0}$
$x - 3 = 0$
$x = 3$

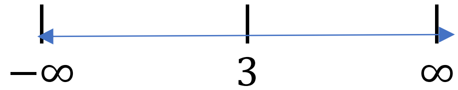
Critical number $x = 4$

3) Plot the critical number(s) on a number line that also includes $-\infty$ *and* ∞.



4) Create interval(s) using only round parenthesis.

$(-\infty, 3)$ $(3, \infty)$



5) Pick a number inside the interval and plug it into the derivative.

Interval: $(-\infty, 3)$
Check $x = 0$
$f"(0) = \frac{4}{(0-3)^3} = 4/-27 = negative\ number$

Interval: $(3, \infty)$
Check $x = 4$
$f"(4) = \frac{4}{(4-3)^3} = 4\ \ positive\ number$

6) Determine whether the graph is concave up or concave down in that interval.
Interval: $(-\infty, 3)$ $concave\ down\ since\ \ f"\ is\ negative$
Interval $(3, \infty)$ $concave\ up\ since\ f"\ is\ positive$

7) Determine if the critical point is an inflection point. A critical number is an inflection point when:

The critical number $x = 3$ is not in the domain of $f(x)$ it can’t be the x-coordinate of an inflection point.

No inflection point

8) Find the y-coordinates of any inflection points.

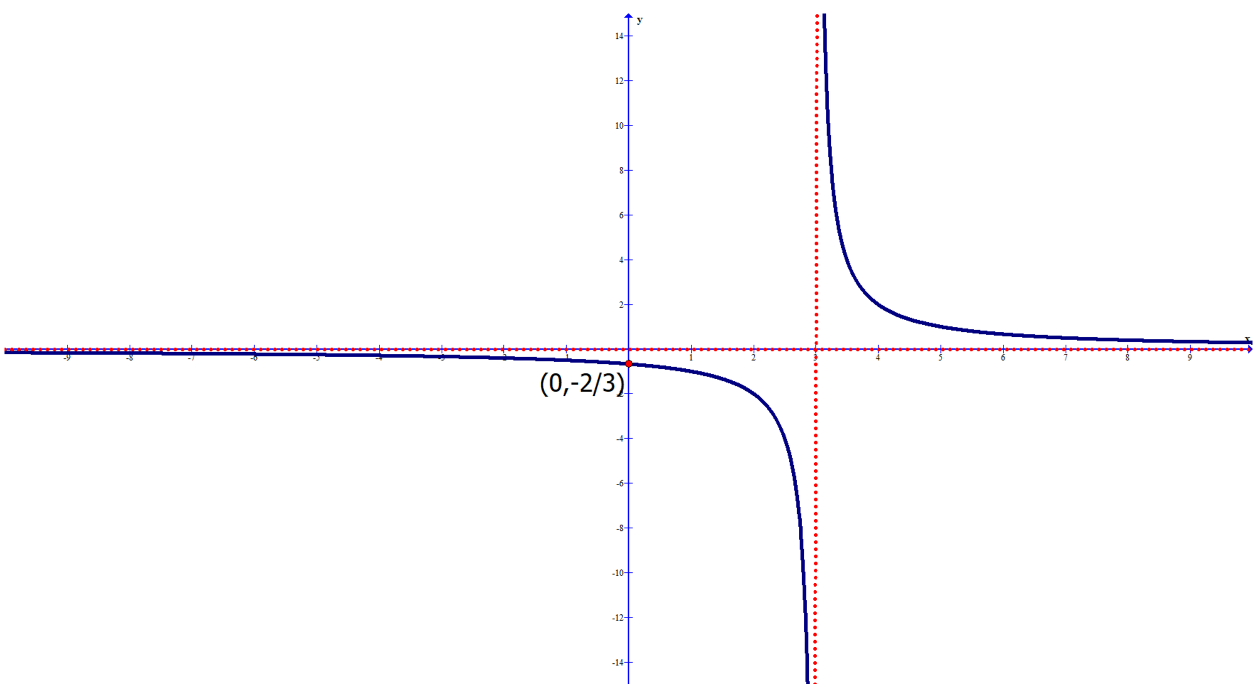
$step\ not\ needed\ as\ there\ is\ no\ inflection\ point$

j) Find the open interval(s) where the function is concave up $(3, \infty)$

k) Find the open interval(s) where the graph of the function is concave down. $(-\infty, 3)$

l) Find all inflection points $none$

m) Sketch a graph (Try your best to create a graph that fits all the criteria. I generally borrow a graph from calculator and make sure each feature I have found is correctly featured on the graph.



Section 3.4 Curve sketching

1) $f(x) = x^3 - 3x^2$

2) $f(x) = 2x^3 - 12x^2$

2a) Find the x-intercept(s), if any

2b) Find the y-intercept, in there is one

2c) Find the interval(s) where the graph of the function is increasing

2d) Find the interval(s) where the graph of the function is decreasing

2e) Find all relative maxima

2f) Find all relative minima

2g) Find the interval(s) where the graph of the function is concave up (if any)

2h) Find the interval(s) where the graph of the function is concave down (if any)

2i) Find all inflection points (if any)

2j) Sketch a graph

2a) Find the x-intercept(s), if any $(0,0)\ and\ (6,0)$

2b) Find the y-intercept, in there is one $(0,0)$

2c) Find the interval(s) where the graph of the function is increasing $(-\infty, 0) \cup (4, \infty)$

2d) Find the interval(s) where the graph of the function is decreasing $(0,4)$

2e) Find all relative maxima $(0,0)$

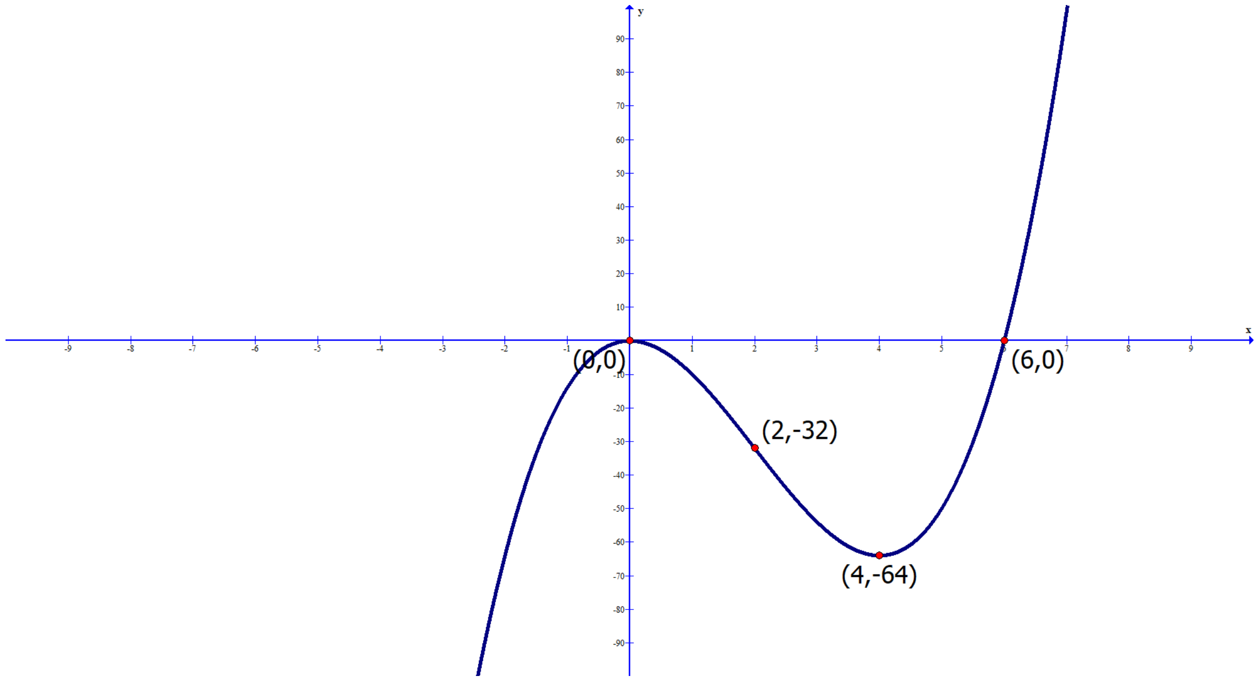
2f) Find all relative minima $(4, -64)$

2g) Find the interval(s) where the graph of the function is concave up (if any) $(2, \infty)$

2h) Find the interval(s) where the graph of the function is concave down (if any) $(-\infty, 2)$

2i) Find all inflection points (if any) $(2, -32)$

2j) Sketch a graph


y
x
(0,0)
(2,-32)
(4,-64)
(6,0)

3) f$(x) = x^3 - 27x$

4) $f(x) = 2x^3 - 54x$

4a) Find the x-intercept(s), if any

4b) Find the y-intercept, in there is one

4c) Find the interval(s) where the graph of the function is increasing
4d) Find the interval(s) where the graph of the function is decreasing
4e) Find all relative maxima
4f) Find all relative minima

4g) Find the interval(s) where the graph of the function is concave up (if any)
4h) Find the interval(s) where the graph of the function is concave down (if any)
4i) Find all inflection points (if any)
4j) Sketch a graph

4a) Find the x-intercept(s), if any $(0,0)\ and\ (\sqrt{27},0)\ (-\sqrt{27},0)$

4b) Find the y-intercept, in there is one $(0,0)$

4c) Find the interval(s) where the graph of the function is increasing $(-\infty,-3) \cup (3,\infty)$

4d) Find the interval(s) where the graph of the function is decreasing $(-3,3)$

4e) Find all relative maxima $(-3,-108)$

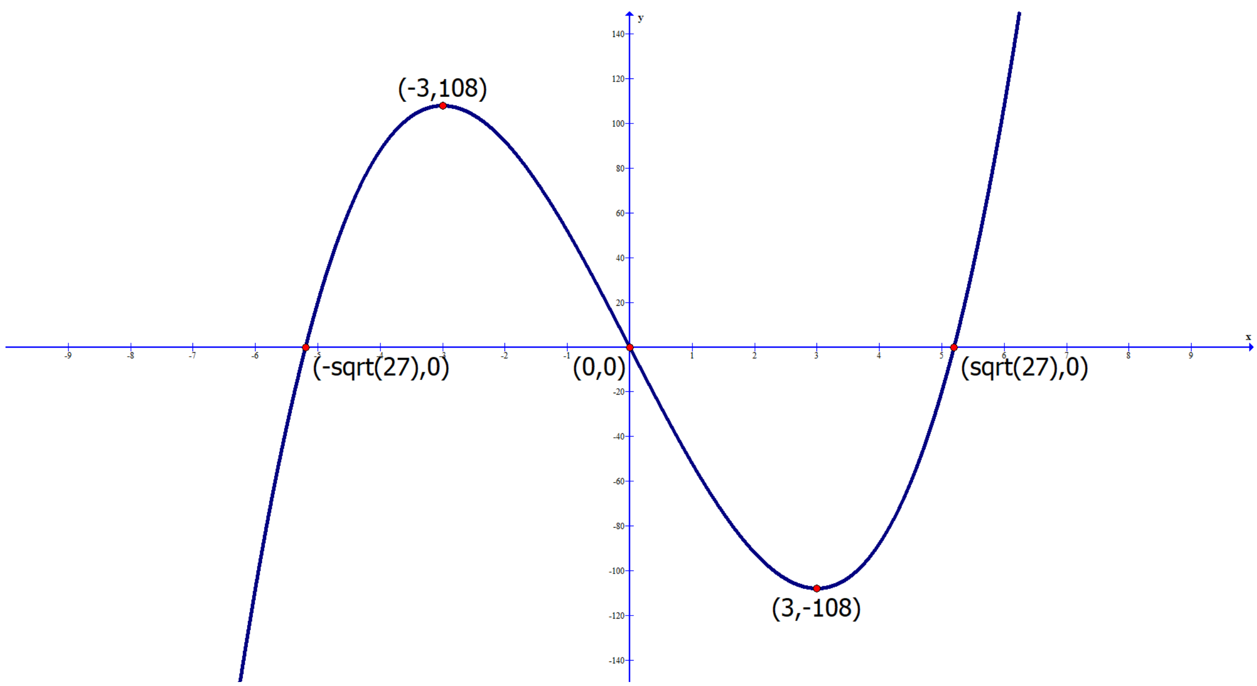
4f) Find all relative minima $(3,-108)$

4g) Find the interval(s) where the graph of the function is concave up (if any) $(0,\infty)$

4h) Find the interval(s) where the graph of the function is concave down (if any) $(-\infty,0)$

4i) Find all inflection points (if any) (0,0)

4j) Sketch a graph



5) $f(x) = x^4 - 4x^3$

6) $f(x) = 2x^4 - 8x^3$

6a) Find the x-intercept(s), if any

6b) Find the y-intercept, in there is one

6c) Find the interval(s) where the graph of the function is increasing

6d) Find the interval(s) where the graph of the function is decreasing

6e) Find all relative maxima

6f) Find all relative minima

6g) Find the interval(s) where the graph of the function is concave up (if any)

6h) Find the interval(s) where the graph of the function is concave down (if any)

6i) Find all inflection points (if any)

6a) Find the x-intercept(s), if any $(0,0)$ *and* $(4,0)$

6b) Find the y-intercept, in there is one $(0,0)$

6c) Find the interval(s) where the graph of the function is increasing $(3, \infty)$

6d) Find the interval(s) where the graph of the function is decreasing $(-\infty, 0) \cup (0,3)$

6e) Find all relative maxima *none*

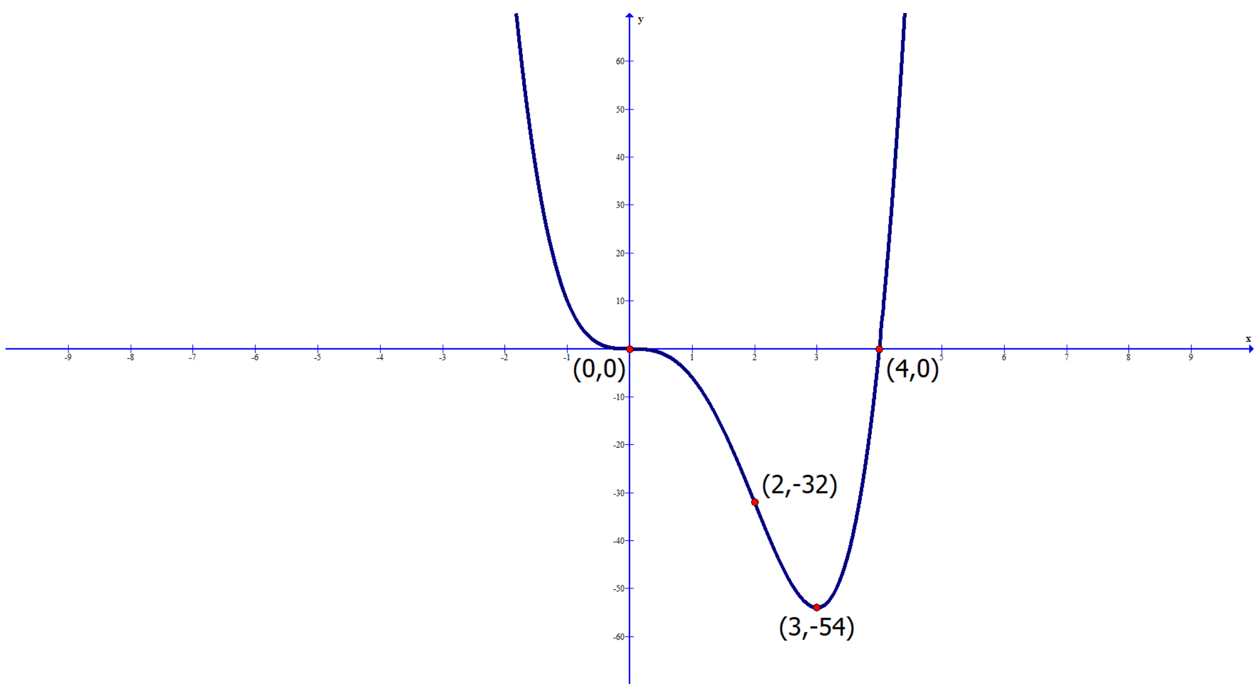
6f) Find all relative minima $(3, -54)$

6g) Find the interval(s) where the graph of the function is concave up (if any) $(-\infty, 0) \cup (2, \infty)$

6h) Find the interval(s) where the graph of the function is concave down (if any) $(0,2)$

6i) Find all inflection points (if any) $(0,0)$ *and* $(2, -32)$

6j) Sketch a graph



7) $f(x) = \frac{3}{x-4}$ Hint: $f'(x) = \frac{-3}{(x-4)^2}$ $f''(x) = \frac{6}{(x-4)^3}$

8) $f(x) = \frac{2}{x-3}$ Hint: $f'(x) = \frac{-2}{(x-3)^2}$ $f''(x) = \frac{4}{(x-3)^3}$

8a) Find the domain

8b) Find the equation of the vertical asymptote

8c) Find the x-intercept(s), if any

8d) Find the y-intercept, in there is one

8e) Find all horizontal asymptotes

8f) Find the interval(s) where the graph of the function is increasing
8g) Find the interval(s) where the graph of the function is decreasing
8h) Find all relative maxima and
8i) Find all relative minima

8j) Find the interval(s) where the graph of the function is concave up (if any)

8k) Find the interval(s) where the graph of the function is concave down (if any)

8l) Find all inflection points (if any)

8a) Find the domain $(-\infty, 3) \cup (3, \infty)$ $or\ \ all\ real\ numbers\ except\ x = 3$

8b) Find the equation of the vertical asymptote $x = 3$

8c) Find the x-intercept(s), if any $none$

8d) Find the y-intercept, in there is one $(0, -\frac{2}{3})$

8e) Find all horizontal asymptotes $y = 0$

8f) Find the interval(s) where the graph of the function is increasing Never

8g) Find the interval(s) where the graph of the function is decreasing $(-\infty, 3) \cup (3, \infty)$

8h) Find all relative maxima and None

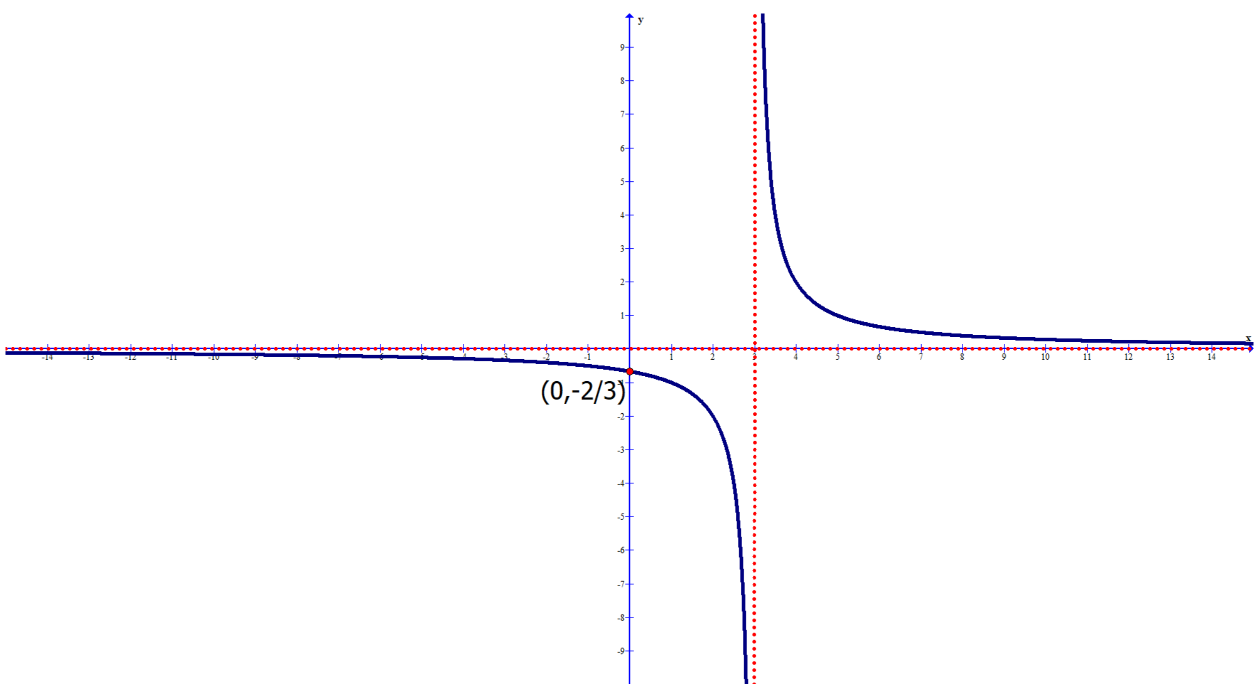
8i) Find a ll relative minima None

8j) Find the interval(s) where the graph of the function is concave up (if any) $(3, \infty)$

8k) Find the interval(s) where the graph of the function is concave down (if any) $(-\infty, 3)$

8l) Find all inflection points (if any) $None$

8m) Sketch a graph



9) $f(x) = \frac{x+2}{x-3}$ Hint: $f'(x) = \frac{-5}{(x-3)^2}$ $f''(x) = \frac{10}{(x-3)^3}$

10) $f(x) = \frac{x+1}{x+5}$ Hint: $f'(x) = \frac{4}{(x+5)^2}$ $f''(x) = \frac{-8}{(x+5)^3}$

10a) Find the domain

10b) Find the equation of the vertical asymptote

10c) Find the x-intercept(s), if any

10d) Find the y-intercept, in there is one

10e) Find all horizontal asymptotes

10f) Find the interval(s) where the graph of the function is increasing

10g) Find the interval(s) where the graph of the function is decreasing

10h) Find all relative maxima and

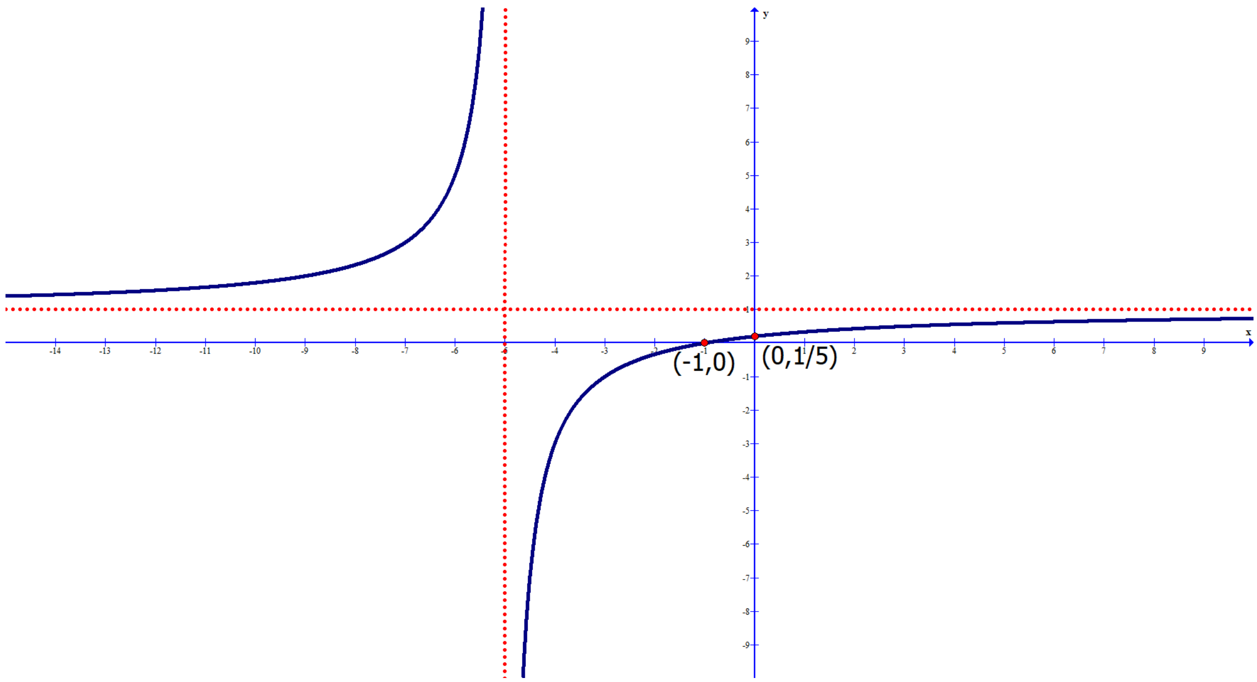
10i) Find all relative minima

10j) Find the interval(s) where the graph of the function is concave up (if any)

10k) Find the interval(s) where the graph of the function is concave down (if any)

10l) Find all inflection points (if any)

10a) Find the domain $(-\infty, -5) \cup$
$(-5, \infty)$ $or\ \ all\ real\ numbers\ except\ x = -5$
10b) Find the equation of the vertical asymptote $x = -5$
10c) Find the x-intercept(s), if any $(-1,0)$
10d) Find the y-intercept, in there is one $(0, \frac{1}{5})$
10e) Find all horizontal asymptotes $y = 1$
10f) Find the interval(s) where the graph of the function is increasing
Never
10g) Find the interval(s) where the graph of the function is decreasing
$(-\infty, -5) \cup (-5, \infty)$
10h) Find all relative maxima and None
10i) Find a ll relative minima None
10j) Find the interval(s) where the graph of the function is concave up (if any) $(-5, \infty)$
10k) Find the interval(s) where the graph of the function is concave down (if any) $(-\infty, -5)$
10l) Find all inflection points (if any) $None$
10m) Sketch a graph



y
x
(-1,0)
(0,1/5)

11) $f(x) = xe^x$

Hint: $f'(x) = e^x(x+1)$ $f"(x) = e^x(x+2)$

12) $f(x) = 3xe^x$

Hint: $f'(x) = 3e^x(x+1)$ $f"(x) = 3e^x(x+2)$

12a) Find the x-intercept(s), if any

12b) Find the y-intercept, in there is one

12c) Find the interval(s) where the graph of the function is increasing
12d) Find the interval(s) where the graph of the function is decreasing
12e) Find all relative maxima
12f) Find all relative minima

12g) Find the interval(s) where the graph of the function is concave up (if any)

12h) Find the interval(s) where the graph of the function is concave down (if any)

12i) Find all inflection points (if any)

12j) Sketch a graph

12a) Find the x-intercept(s), if any $(0,0)$

12b) Find the y-intercept, in there is one $(0,0)$

12c) Find the interval(s) where the graph of the function is increasing $(-1, \infty)$

12d) Find the interval(s) where the graph of the function is decreasing $(-\infty, -1)$

12e) Find all relative maxima $none$

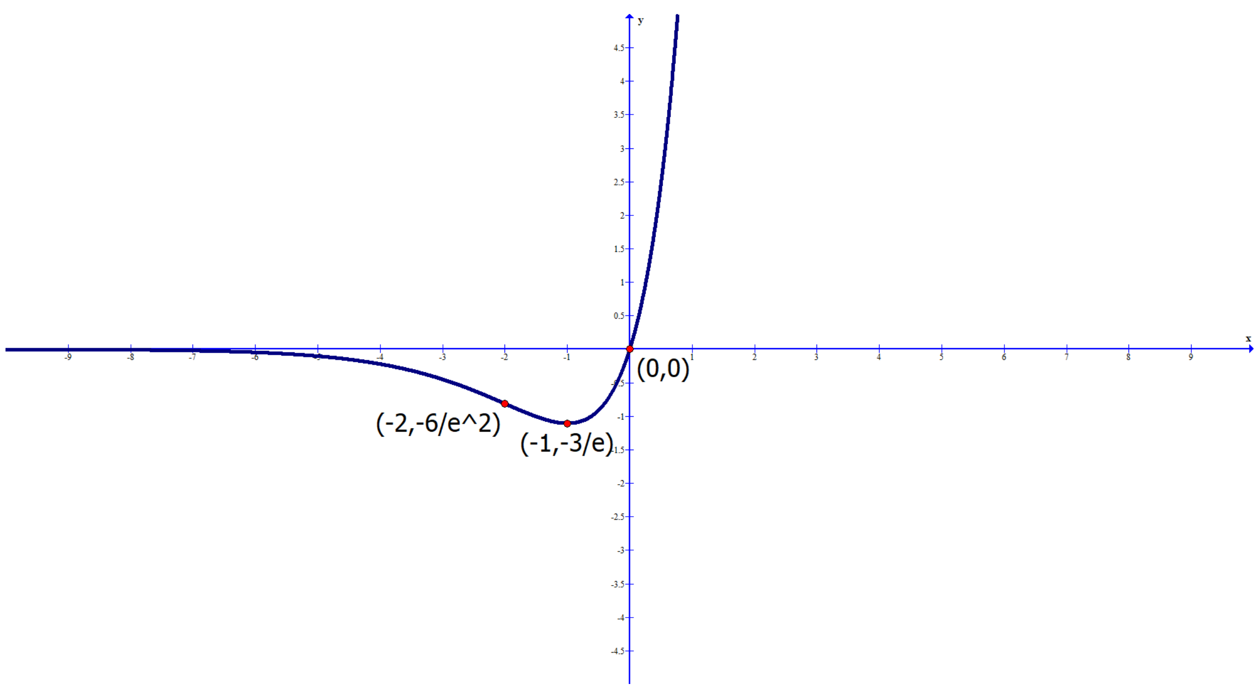
12f) Find all relative minima $\left(-1, \frac{-3}{e}\right)$

12g) Find the interval(s) where the graph of the function is concave up (if any) $(-2, \infty)$

12h) Find the interval(s) where the graph of the function is concave down (if any) $(-\infty, -2)$

12i) Find all inflection points (if any) $\left(-2, \frac{-6}{e^2}\right)$

12j) Sketch a graph



13) $f(x) = 2xe^x$

Hint: $f'(x) = 2e^x(x+1)$ $f"(x) = 2e^x(x+2)$

14) $f(x) = 4xe^x$

Hint: $f'(x) = 4e^x(x+1)$ $f"(x) = 4e^x(x+2)$

14a) Find the x-intercept(s), if any

14b) Find the y-intercept, in there is one

14c) Find the interval(s) where the graph of the function is increasing
14d) Find the interval(s) where the graph of the function is decreasing
14e) Find all relative maxima
14f) Find all relative minima

14g) Find the interval(s) where the graph of the function is concave up (if any)

14h) Find the interval(s) where the graph of the function is concave down (if any)

14i) Find all inflection points (if any)

14a) Find the x-intercept(s), if any $(0,0)$

14b) Find the y-intercept, in there is one $(0,0)$

14c) Find the interval(s) where the graph of the function is increasing $(-1,\infty)$

14d) Find the interval(s) where the graph of the function is decreasing $(-\infty,-1)$

14e) Find all relative maxima $none$

14f) Find all relative minima $\left(-1,\frac{-4}{e}\right)$

14g) Find the interval(s) where the graph of the function is concave up (if any) $(-2,\infty)$

14h) Find the interval(s) where the graph of the function is concave down (if any) $(-\infty,-2)$

14i) Find all inflection points (if any) $\left(-2,\frac{-8}{e^2}\right)$

14j) Sketch a graph